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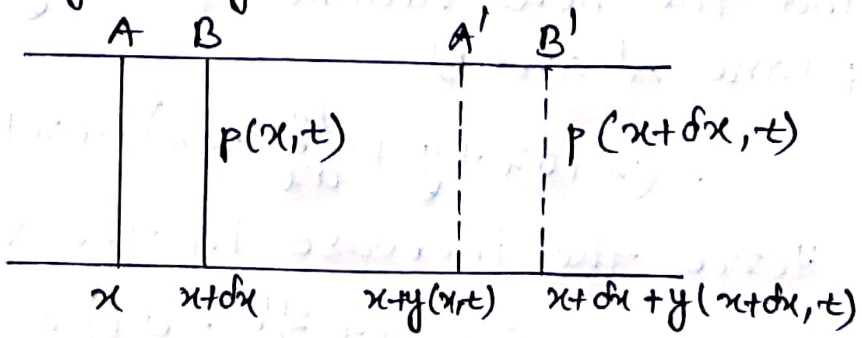
paper-1

Velocity of Longitudinal (Sound) wave in a fluid medium.

When a plane progressive longitudinal sound wave travels in a gas, its particles execute a simple harmonic motion along the line of propagation with the same period and amplitude, but with a continuously varying phase. The separation between the successive particles of the gas changes in such a way that at any instant, the same particles are alternately crowed together and spread out, producing the travelling waves of compression and refraction along the direction of propagation. Therefore, the pressure varies from particle to particle all through the gaseous medium.

Consider a cylindrical tube of gas of unit area of cross-section, and two plane section A and B at right angles to the axis of the tube.



Let the undisturbed positions of these planes measured along the axis of the tube from some arbitary origin be x and $x+\delta x$ respectively.

Let a plane sound wave pass along the axis of the tube and at any instant, the displacement of the plane A be y, and A' be the displaced position of the plane A distant $(x+y)$ from the arbitary origin.

Thus the rate of change of displacement with respect to distance $x = \frac{dy}{dx}$

$\therefore$ The displacement of the plane B will be $= \left(y+\frac{dy}{dx}, \delta x\right)$ Hence the displaced position B' of this plane is

$$= \left(x+\delta x + y + \frac{dy}{dx}, \delta x\right)$$

Thus we find that the passage of wave has resulted in a change in volume of the gas enclosed between the planes A and B and hence a variation of pressure takes place from point to point in the gas along the axis of the tube.

original volume of the gas between the planes A and B

$$= (x+\delta x) - \delta x = \delta x$$

and the new volume of the gas between the plane A' and B'

$$= \left(x+\delta x + y + \frac{dy}{dx}.\delta x\right) - (x+y) = \left(\delta x + \frac{dy}{dx}.\delta x\right)$$

Hence the increase in the volume of the gas

$$= \left(\delta x + \frac{dy}{dx} - \delta x\right) = \frac{dy}{dx}.\delta x$$

$\therefore$ volume strain $= \dfrac{\text{change in volume}}{\text{original volume}}$

$$= \frac{\left(\frac{dy}{dx}\right)\cdot \delta x}{\delta x} = \frac{dy}{dx}$$

Let p be the additional pressure on the plane A (now displaced to A') above its normal pressure.

The Corresponding pressure on the plane B (now displaced to B') $= p + \frac{dp}{dx}\cdot \delta x$

The directions of these pressures acting on the two planes A' and B' are shown in the diagram and they are equivalent to

(i) Equal and opposite pressure p, p acting on the column of gas enclosed between the two planes, which Constitute the stress and produced the volume strain $\frac{dy}{dx}$ in the sample of the enclosed gas.

(ii) A resultant pressure increase $\frac{dy}{dx}\cdot\delta x$ in the direction B' to A', which constitutes the restoring force producing an acceleration in the sample of gas in the direction form B' to A', causing it to return.

Let K be the bulk modulus or volume elasticity of the gas. Then by Hook's law, we have

$$K = \frac{\text{stress}}{\text{strain}} = -\frac{p}{dy/dx}$$

$$\text{or,} \quad p = -K\frac{dy}{dx}$$

$-$ve sign shows that an increase in pressure decrease in volume of the gas

The restoring force acting on the Sample of gas enclosed between the two planes is $= \frac{dp}{dx} dx$

putting the value of p from above equation

Restoring force $= \frac{d}{dx}\left(-K\frac{dy}{dx}\right) dx$

$$= -K\frac{d^2y}{dx^2} \cdot dx$$

If ρ be the density of the gas, then

mass of the ~~ed~~ enclosed gas = density × volume

$$= \rho \times dx$$

If $-\frac{d^2y}{dx^2}$ be the instantaneous acceleration acting on the Sample of gas from B' to A', then

The resultant force acting on the Sample of gas from B' to A'

$$= \rho \times dx\left(-\frac{d^2y}{dt^2}\right)$$

The equation of motion will be

$$-K\frac{d^2y}{dx^2} \cdot dx = \rho \times dx\left(-\frac{d^2y}{dt^2}\right)$$

or $$\frac{d^2y}{dt^2} = \frac{K}{\rho} \cdot \frac{d^2y}{dx^2}$$

Comparing above equation with the general differential equation of wave motion

$$\frac{d^2y}{dt^2} = v^2\frac{d^2y}{dx^2}$$, we get

$$v^2 = \frac{K}{\rho}, \quad \text{or} \quad \boxed{v = \sqrt{\frac{K}{\rho}}}$$

This is the expression for the velocity of a plane Simple harmonic Sound wave in a fluid medium.